

Enhanced Gauge Symmetry in M(atrix) Theory

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We discuss the origin of enhanced gauge symmetry in ALE (and K3) compactification of M theory, either defined as the strong coupling limit of the type IIa superstring, or as defined by Banks et al. In the D-brane formalism, wrapped membranes are D0 branes with twisted string boundary conditions, and appear on the same footing with the Kaluza-Klein excitations of the gauge bosons. In M(atrix) theory, the construction appears to work for arbitrary ALE metric.

1. Introduction

Recently Banks et. al. have proposed a definition of eleven-dimensional M theory in the infinite momentum frame, as a large N limit of a supersymmetric matrix quantum mechanics [1]. To support this, they start with the fact that this quantum mechanics describes the D0-branes which dominate the strong coupling limit of IIA string theory, argue that anti D0-branes decouple in the IMF, and then adapt results of [2] showing that the theory can reproduce supergravity interactions without need of the original closed strings. From this point of view, modifications to the background can be made by adapting the corresponding modifications to the IIA string, providing definitions of the five-brane [3] and toroidal compactifications [1,4,5]. Not all physics follows from IIA arguments however and a quite non-trivial non-IIA result is the appearance of the supermembrane with correct physics [6,7,1,8].

In this note we study another example of IIA-derived M-theory physics; the enhanced gauge symmetry of compactifications on $K3 \times \mathbb{R}^7$ in the orbifold limit [9] which follows from the proposal by Hull and Townsend of strong-weak coupling duality between IIA on $K3$ and the heterotic string on T^4 [10]. Its M theory origin is clear – membranes wrapped on small supersymmetric two-cycles become particles with conventional (vector) gauge charge in the dimensionally reduced theory, and when such two-cycles degenerate to zero volume, these particles include massless gauge bosons.

Since this phenomenon is local, we can see it by formulating the theory in the neighborhood of the degenerating two-cycles, in other words on $\mathcal{M}_\zeta \times \mathbb{R}^7$, where \mathcal{M}_ζ is an ALE space asymptotic to \mathbb{C}^2/Γ . Γ is a finite subgroup of $SU(2)$ and it has an associated simply laced extended Dynkin diagram \mathcal{G} , an affine Lie algebra \hat{G} and finite Lie algebra G [11]. The enhanced gauge symmetry obtained by maximal degeneration to the singularity \mathbb{C}^2/Γ is simply G .

An explicit hyperkähler quotient construction of \mathcal{M}_ζ with its metric was made by Kronheimer [12], and this construction appears in D-brane physics: the natural construction of D-branes embedded at a point in an orbifold produces a gauge theory whose moduli space is \mathcal{M}_ζ [13]. We will use this construction for D0-branes in IIA string theory.

A wrapped D2-brane also has a known D-brane realization in this construction [14,15]: it is a D0-brane with twisted boundary conditions for the open strings, which project out the moduli moving it from the fixed point. An easy computation shows that it is charged under twist sector RR fields, and since the wrapped membrane is the only charged BPS state in the large volume limit, the two objects must be continuously connected. Its non-zero mass is interpreted as a consequence of an implicit $B \neq 0$ of the orbifold construction [16]; we will be able to determine B for any Γ .

This B corresponds to a Wilson line in the additional dimension of M theory, and thus the Kaluza-Klein states of massless gauge bosons and the massive gauge bosons of spontaneously broken gauge symmetry appear on the same footing in this construction.

Taking this construction for D0-branes and adapting it according to the rules of [1] provides a construction of M theory on $\mathcal{M}_\zeta \times \mathbb{R}^7$. There are several differences with the string theory discussion. First, the moduli of the ALE are controlled by Fayet-Iliopoulos terms in the gauge theory. While these were derived in [13] as couplings to closed string twist fields, here they are postulated. This is appropriate as we are discussing different backgrounds in the infinite momentum frame, which should be realized by changing parameters in the Lagrangian. Second, the expectation value of B disappears in the limit, and the full enhanced gauge symmetry appears. Finally, there appears to be no analog of the upper bound on the blow-up parameter ζ at the string scale which follows from the general results of [2].

Another test can be made by introducing a five-brane wrapped on K3 or M_ζ . This produces the heterotic string which dominates the strong coupling limit, and we must see a level 1 action of \hat{G} on the spectrum of this string.

As in [3], we define the five-brane by introducing a vector hypermultiplet into the D0-brane quantum mechanics. This theory is the dimensional reduction of the general theory of [13], corresponding to the hyperkähler quotient construction of instanton moduli space of Kronheimer and Nakajima [17]. Following Harvey and Moore [18], if we assume that the space of bound states is the sheaf cohomology of this moduli space, then results of Nakajima [19] imply the existence of these bound states as well as the \hat{G} action.

2. D0-branes on orbifolds

D-branes on an orbifold are defined as in [13]; we take the $U(N)$ gauge theory of N D-branes at the fixed point in \mathbb{C}^2 and quotient by a combined action of Γ on space-time and the Chan-Paton factors: $A_\mu = \gamma_{CP} A_\mu \gamma_{CP}^{-1}$ and $Z^i = \gamma_j^i \gamma_{CP} Z^j \gamma_{CP}^{-1}$. The derivation can be made for 5-branes and the result is a $\mathcal{N} = 1$, $d = 6$ gauge theory whose Lagrangian (at leading order in α' , which is the only part used in [1]) is determined by the choice of gauge group and matter representation; this is given in [13] for the A series, and in [20] for the D and E series. The quantum mechanics of D0-branes is its dimensional reduction.

The field content is determined by a choice of Γ representation R . Let the irreducible representations of Γ be R_i with $0 \leq i \leq \text{rank } G$ and their dimensions be n_i . R_0 is the trivial representation, R_1 the fundamental (the same as the action on \mathbb{C}^2), and the R_i are associated with the extended Cartan matrix \hat{C} by the McKay correspondence,

$$R_1 \otimes R_i = \oplus_j (2\delta - \hat{C})_{ij} R_j. \quad (2.1)$$

In terms of the extended Dynkin diagram \mathcal{G} , each node is an irrep R_i , and the non-zero terms in (2.1) are links. If R decomposes as

$$R = \sum_{i=0}^r v_i R_i, \quad (2.2)$$

the resulting gauge symmetry is $\prod_i U(v_i)$, and each link of \mathcal{G} comes with a hypermultiplet in the (v_i, \bar{v}_j) . Besides the overall coupling, the parameters of the Lagrangian are an $SU(2)_R$ triplet of Fayet-Iliopoulos terms for each of the $U(1)$ factors, ζ_i^n with $1 \leq n \leq 3$, $0 \leq i \leq r$ and $\sum_i \zeta_i^n = 0$. These determine the periods of the metric on the hyperkähler quotient and in string theory are determined by expectations of twist fields [13].

The simplest case to interpret is N copies of the regular representation, $v_i = N n_i$. The Higgs branch of the moduli space for generic ζ is the symmetric product $(\mathcal{M}_\zeta \times \mathbb{R}^5)^N / S_N$, the positions of N independent D0-branes in space-time. Following [9], these are interpreted as Kaluza-Klein states of M theory on $\mathcal{M}_\zeta \times \mathbb{R}^7$ with momentum $p_{11} = 1/R_{11}$. Consistency of this interpretation predicts bound states with $p_{11} = N/R_{11}$ corresponding to all partitions of this momenta among up to N particles.

Along with the bulk supergravity fields, we expect additional bound states localized near the fixed point; in particular the BPS states which come from the decomposition

$$C^{(3)} = \sum_i A_\mu^{(i)}(x) \omega^{(i)}, \quad (2.3)$$

where $\omega^{(i)}$ are the normalizable harmonic forms on the ALE, and their supersymmetry partners forming a full gauge multiplet.

The simplest case in which to check this is $\Gamma = \mathbb{Z}_2$, $v_1 = v_2 = 1$. This is $U(1)^2$ gauge theory, but the diagonal $U(1)$ decouples, and the non-trivial dynamics is that of $U(1)$ gauge theory coupled to two hypermultiplets of charge 2. This is a system for which the arguments of [21,22,2] establish the existence of bound states; it is identical (up to the unit of charge) to the system of a D0-brane in the presence of two D4-branes and using string duality, has the same bound states as two D0-branes and a single D4-brane. Thus one predicts a “new” bound state and an additional set of BPS states in the symmetric product of two single 0–4 bound states. The new bound state will be interpreted as the $p_{11} = 1/R$ KK mode of the $U(1)$ gauge multiplet, while the others could be interpreted as the product of a bound state with $(v_1, v_2) = (1, 0)$ and one with $(v_1, v_2) = (0, 1)$.

Now there was no consistency condition in the string theory requiring all v_i equal, and the interpretation of more general states is briefly described in [14] (it was known to the authors of [13] but not mentioned there): taking a single $v_i = 1$ and the rest zero produces

a D2-brane wrapped around a non-trivial two-cycle of the ALE. In string theory, the test of this is to check that it is a source of twist sector RR field, the orbifold realization of the fields (2.3). This is shown in the appendix to [13], for $\Gamma = \mathbb{Z}_n$.

The representations R_i are associated with homology two-cycles σ_i in \mathcal{M}_ζ , and R_0 is associated with $\sigma_0 = -\sum_i n_i \sigma_i$. The intersection form $\langle \sigma_i \cup \sigma_j \rangle$ is the extended Cartan matrix \hat{C}_{ij} [12]. This leads to a further association of R_i and σ_i with the simple roots $\vec{\alpha}_i$ and the lowest root $\vec{\alpha}_0$ of G ; these translate directly into the charges $\vec{Q}_i = \vec{\alpha}_i$ of the $r + 1$ elementary wrapped two-branes.

The coupling to the untwisted sector is universal, so all of these D0-branes have mass $1/c_2 g_s$, where $c_2 = \sum_i n_i$ is the Coxeter number of Γ . This mass is also determined by the central charge formula [10] to be

$$m = \frac{1}{g_s} \min_{n \in \mathbb{Z}} \left| \int_{\vec{Q} \cdot \vec{\sigma}} B + iJ + n \right| \quad (2.4)$$

where J is the Kähler form with respect to the complex structure for which σ is a holomorphic curve (i.e. with $\int \Omega = 0$). Matching this for $\zeta = 0$ determines the background B for the orbifold. * For \mathbb{Z}_2 , it is $B = 1/2$ as found in [16]. For \mathbb{Z}_n , the cycles $\sigma_i, i > 0$ associated with simple roots have $\int B = 1/n$. This corresponds to a non-zero $C_{11\mu\nu}$ in eleven dimensions and thus to an $SU(n)$ Wilson line

$$A_{11} = \frac{1}{R_{11}} \begin{pmatrix} \frac{\sigma}{n} & 0 & 0 & \dots & 0 \\ 0 & \frac{\sigma+1}{n} & 0 & \dots & 0 \\ 0 & 0 & \frac{\sigma+2}{n} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \frac{\sigma-1}{n} \end{pmatrix} \quad (2.5)$$

where $\sigma = \frac{n-1}{2}$. There is a strong analogy with the Wilson line breaking E_8 to $SO(16)$ in the relation of type I' string theory to M theory [23]. Perhaps there is a general rule determining such symmetry breakings.

Thus the $\sum v_i = 1$ states provide the gauge bosons corresponding to simple roots of G . They fall into $8 + 8$ component supermultiplets; more explicitly the orbifold projection retains half of the 16 components of the gaugino, transforming in the doublet of the $SU(2) \subset SO(4)$ which is singlet under the orbifold projection and the 4 of $SO(5)$. These

* The dependence of the mass of a finite wrapped D2-brane on $B \bmod 1$ is realized by the additional dependence on a world-volume gauge field F and the possibility of $\int F \neq 0$. In the present definition, it appears to be reflected in subtleties in the D0-brane coupling to the twist sector B similar to those found in the appendix of [13].

act on a multiplet whose bosons are a vector of $SO(5)$ and three scalars, the physical states of a $d = 7$ gauge boson and the metric fluctuations.

Their bound states must provide all of the gauge bosons, and thus we predict that a new supermultiplet of bound states exists for each root α and integer N , with $p_{11} = (N + \frac{1}{2}B \cdot \alpha)/R_{11}$; in other words for each sector with $(\sum_i v_i \alpha_i)^2 = 2$. This must be true in the full IIA theory for consistency of M theory; the explicit bound state we described lends support to the conjecture that these are bound states in the pure D0 quantum mechanics.

Although enhanced gauge symmetry is spontaneously broken, it is fairly manifest. In the D-brane realization, supergravity Kaluza-Klein states and wrapped membrane states appear on an equal footing.

Turning on the moduli ζ_i^n modifies the gauge symmetry breaking (they correspond to Wilson lines A_n in the T^3 of the dual heterotic string) and the effective Hamiltonian. For $|\zeta| \ll \langle B \rangle$, (2.4) has the expansion

$$\begin{aligned} m &= \frac{1}{g_s} \sqrt{B^2 + \zeta^2} \\ &= \frac{B}{g_s} + \frac{\zeta^2}{2Bg_s} + O(\zeta^4) \end{aligned} \tag{2.6}$$

where $\zeta^2 = \sum_n (\vec{Q} \cdot \vec{\zeta}^n)^2$. The $O(\zeta^2)$ dependence can be seen explicitly for $\sum v_i = 1$ in the D-term potential, which degenerates to $V \propto \sum_n (\zeta^n)^2$. For $\zeta \sim 1$, stringy corrections are known to be important [13].

2.1. From M theory to M(atrix) theory

Following [1], we now regard this system as the definition of M theory on \mathcal{M}_ζ in a sector with longitudinal light-cone momentum $P_- = N/R$, and take the $R_{11} \rightarrow \infty$ limit. The Wilson line (2.5) disappears, and the massless charged gauge bosons at $\zeta = 0$ are manifest. Now it is this observation which confirms the identification of $\sum v_i = 1$ states as wrapped membranes.

It is important to check the basic tenets of [1] in this context, for example that supergravity interactions between these particles are correctly reproduced by quantum open string effects. This issue will be discussed in [24].

For any $|\zeta| \gg l_{p11}^2$, the classical analysis of the resulting Higgs branch appears to be valid, meaning there would be no restriction on the blow-up parameter in this construction. Consistent with this, the D-term potential exactly reproduces the term $m^2/p_{11} = \zeta^2/p_{11}$ in the IMF 0-brane energy.

The construction must work for all states, not just BPS states. There are clear predictions for the states on the blowup with $\zeta \gg l_p^2$, where the conventional supergravity

analysis is valid: we diagonalize the basic supergravity and membrane Hamiltonians to get higher modes in the KK expansion (2.3), and local excitations of the wrapped membranes. However, estimating their couplings to the bulk states using the known membrane coupling $\int h_{\mu\nu} \partial X^\mu \partial X^\nu$ leads to the conclusion that they are unstable and thus only the full dynamics can be sensibly compared, a very interesting open problem.

3. Five-branes

We add a five-brane as in [3], by adding vector degrees of freedom. Different orientations will have different physics. We can put it at a point in the ALE (by starting off with images), and get a theory which should contain “tensionless strings” in the limit $\zeta = X = 0$. Seeing these should be quite interesting but requires knowing how to construct membranes ending on the five-branes.

If we instead embed the longitudinal dimensions in an ALE, we get a piece of the five-brane wrapped around K3. This is the heterotic soliton which dominates the small K3 limit. We are treating the large K3 limit, but we must see BPS states of this soliton in any case. These are excitations of the bosonic left movers admitting unbroken $(0, 4)$ supersymmetry and the action of world-sheet current algebra, affine \hat{G} at level 1. This symmetry is broken both by $\zeta \neq 0$ and, at finite R_{11} , by the Wilson line (2.5), but this will be realized by explicit terms in the Hamiltonian.

Physical five-brane degrees of freedom are new bound states of zero-branes. In the IMF, we identify the left and right world-sheet stress tensors with

$$\begin{aligned} H = L_0 &= \frac{1}{2}(p - w)^2 + N_0 \\ P_{11} = \bar{L}_0 &= \frac{1}{2}(p + w)^2 + \bar{N}_0. \end{aligned} \tag{3.1}$$

Note that we are not restricted to $L_0 = \bar{L}_0$, because we are considering a finite piece of an infinite string.

The choice of which chirality has world-sheet supersymmetry is determined by the chirality of the additional vector degrees of freedom. If we make this compatible with the unbroken supersymmetry on the orbifold, we get non-trivial supersymmetries commuting to produce H , so L_0 is the supersymmetric side (say right movers) and BPS excitations can have non-zero \bar{N}_0 . If we make the other choice, the supersymmetry becomes trivial and \bar{L}_0 is the supersymmetric side. In this case, BPS states will not be realized as bound states of D0-branes.

The full gauge theory is now a D0–D4 brane system, also derived in [13] (section 5). These theories are parameterized by a set of non-negative integers w_i where $\sum w_i$ is the total number of D4-branes. They are obtained from the pure D0-brane theories by adding w_i hypermultiplets in the fundamental representation of $U(v_i)$ for each i . As shown in [17] (and reviewed in section 9 of [13]), the Higgs branch of moduli space is generally equivalent to a moduli space of instantons in the D4-brane gauge theory. The choice of w_i translates into a choice of first Chern class in this language; a single heterotic string would have a single $w_k = 1$, with k denoting a choice of sector in the world-sheet theory.

It is natural to look for D0–D4 bound states in the supersymmetric quantum mechanics on this moduli space, and thus identify them with elements of the moduli space cohomology. Of course the moduli space approximation is not exact and furthermore these spaces are typically singular. Harvey and Moore [18] discuss some of the issues here, and propose that the general identification will be between the Hilbert space of bound states and the complex cohomology of the moduli space of coherent simple sheaves. This generalization is particularly significant in the present case of a single D4-brane, as “ $U(1)$ self-dual instantons” are at best rather singular objects.

Existing results on bound states in quantum mechanics along with the string duality arguments of [18] all support this identification, and we will assume it here. This allows us to make use of the results of Nakajima [19] on the cohomology and especially the celebrated Kac-Moody algebra which acts on the cohomology. For our present case of $\sum_i w_i = 1$, this will be a \hat{G} action at level 1. The generators of this algebra E_i , F_i and H_i are as follows: the Cartan subalgebra H_i acting on a cohomology class of the sector of moduli space characterized by integers v_i has eigenvalue v_i ; the operators E_i add a single twisted D0-brane (and thus increase v_i by one); the operators F_i are their conjugates. These are effectively ‘second quantized’ operators and their natural physical interpretation is in terms of Harvey and Moore’s “correspondance conjecture” [18], defining their action on the BPS states.

We claim that this is the standard world-sheet current algebra which acts on the spectrum of a single heterotic string. One test of this is that the left-moving Virasoro generators \bar{L}_n must contain the Sugawara stress-tensor as one component. This requires that P_{11} as defined in M theory, i.e. $(\sum v_i + c_k)/c_2 R_{11}$ where c_k is a constant possibly depending on the sector k , be equal to the Sugawara L_0 . This follows from Nakajima’s results, which make L_0 the second Chern class of the sheaf. The constant c_k is a contribution from the non-zero first Chern class present for $w_i > 0$, $i \neq 0$.

Nakajima’s results also support this identification of the spectrum – in particular, it is shown that the cohomology contains all highest weight representations – but we have not

verified that the full cohomology is isomorphic to the spectrum of BPS states. Following Harvey and Moore, this must follow from IIa – heterotic string duality, because the D0 and D4 branes of the construction are sensible objects in the IIa string. Indeed, the present discussion differs from theirs mainly in that we are considering a state containing an infinitely long heterotic string rather than perturbative heterotic string states.

4. Conclusions

In this note we showed that Dirichlet branes on orbifolds provide a simple and explicit way to see the enhanced gauge symmetry of the IIa string and M theory on K3. The construction produces an explicit realization of world-sheet current algebra for the wrapped five-brane which becomes the dual heterotic string. Although these are not first quantized operators (they change the D0-brane number), it should be possible to describe their action fairly explicitly.

In principle, the same operators adding and removing zero-branes act on the space of pure 0-brane bound states. They will realize the subgroup of global gauge transformations and it might be (extrapolating beyond Nakajima’s results) that this is contained in a Kac-Moody algebra at level zero. The natural interpretation of such an algebra in M theory (with X^{11} space-like, so before going to the IMF) would be the subgroup of gauge transformations with X^{11} dependence. Such an interpretation would imply that the Kac-Moody action can be extended to all states, not just BPS states.

The Virasoro algebra associated with Nakajima’s Kac-Moody algebra is of course part of the Virasoro algebra which plays the key role in the Lorentz invariance of the light-cone heterotic string. We believe that extending this action to the full state space will be a key element in understanding the Lorentz invariance not manifest in the treatment of [1].

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